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THE ACTIVITY COEFFICIENTS OF POTASSIUM CHLORIDE. AN APPLICATION OF THE EXTENDED DEBYE-HÜCKEL THEORY TO INTERPRETATION OF FREEZING POINT MEASUREMENTS

By Hugh M. Spencer

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Introduction

The importance of activity coefficients of potassium chloride solutions to questions of electrode potentials has made desirable a reinterpretation of the values hitherto available. The more recent values are by Scatchard¹ (1925, based (1) on e. m. f. measurements of MacInnes and Parker 1925, and of Beattie 1920, (2) on freezing point measurements, not corrected for heats of dilution, (3) on vapor pressure measurements of Lovelace, Frazer and Sease at 20°), Randall and White (1926, based on vapor pressure measurements of Lovelace, Frazer and Sease interpreted with the help of freezing point measurements), Jones and Bury (1927, based on their freezing point measurements, corrected for the heats of dilution)² and Harned (1929, based on e. m. f. measurements of concentration cells with intermediate amalgam electrodes).

Though the Hückel equation

$$\log \gamma = -\frac{0.356 \sqrt{2c}}{1+A \sqrt{2c}} + 2Bc - \log(1+0.036m) \tag{1}$$

certainly seems to serve admirably as an empirical expression for representation of activity coefficients as a function of concentration for solutions of moderate and high concentration, the assumptions upon which the equation was derived are no longer valid at these concentrations, and in fact at lower concentrations where the assumptions are probably valid³ the extended Debye–Hückel theory for symmetrical valence type salts leads to another form⁴

¹ (a) G. Scatchard, THIS JOURNAL, **47**, 648 (1925); (b) M. Randall and A. McL. White, *ibid.*, **48**, 2514 (1926); (c) E. R. Jones and C. R. Bury, *Phil. Mag.*, [7] **3**, 1032 (1927); (d) H. S. Harned, THIS JOURNAL, **51**, 416 (1929).

² Some gross error was introduced in correcting for the heat of dilution, resulting in too large corrections.

³ T. H. Gronwall, V. K. La Mer and K. Sandved, Physik. Z., 29, 358 (1928).

⁴ The first terms of Equations 1 and 2 are the same. Harned found for potassium chloride A = 0.76, B = 0.0204 and hence a = 3.26 Å. (This differs from the value selected in the present paper.) For 0.01 M KCl, by Equation 1, log $\gamma = 0.04528 + 0.00041 - 0.00016 = -0.04503$; $\gamma = 0.9015$ and by Equation 2, log $\gamma = -0.04528 - 0.00057 - 0.00003 = -0.04588$; $\gamma = 0.8997$. Thus the terms added by the complete expansion of the Debye-Hückel equation have the same sign as the limiting effect term whereas the net value of the added terms of the Hückel equation has opposite sign.

$$-\log \gamma = 1.53636 \left(\frac{z^2}{10^8 a}\right) \frac{x}{1+x} - 0.15382 \left(\frac{z^2}{10^8 a}\right)^3 10^3 \left[\frac{1}{2} X_3(x) - 2 Y_3(x)\right] - 0.0770 \left(\frac{z^2}{10^8 a}\right)^5 10^5 \left[\frac{1}{2} X_6(x) - 4 Y_6(x)\right]$$
(2)

These factors as well as the uncertainty that the cell reactions in the amalgam cells were purely the postulated ones made it appear unwise to accept the determination made by use of Equation 1. No satisfactory calculations of the activity coefficients from freezing point measurements with corrections for the heats of dilution have been made. The partial molal heats of dilution and the partial molal heat capacities necessary for such calculations have been determined by Lange and Monheim, Wüst and Lange and by Randall and Rossini, respectively.⁵ The freezing point determinations used include the recent ones of Karagunis, Hawkinson and Damköhler.⁶

These data have been treated according to the methods of Lewis and Randall and of Randall and White⁷ with an added feature. Since the uncertainty in measurements of $j/m^{1/4}$ rapidly becomes very large as the dilution is increased as does that of j/m for non-electrolytes,⁸ some guiding principle besides the theoretical limit at m = 0, was sought and found in theoretical values of j calculated by Gronwall, La Mer and Sandved's³ extension of the Debye–Hückel theory.

Theoretical $j/zm^{1/2}$ and Experimental $j/m^{1/2}$ for Potassium Chloride.— Values of $j/m^{1/2}$ were calculated⁹ for each set of experimental data and plotted as ordinate versus $m^{1/2}$ as abscissas. The values from the work of Jones and Bury, Karagunis, Hawkinson and Damköhler, Adams, Hovorka and Rodebush and Menzel fall on a smooth curve mostly within the limits of error caused by the form of the function. Even though the limiting value of $j/m^{1/2}$ be used, an undesirable amount of uncertainty remains in the course of the curve between the point where the form of the function makes its experimental value too uncertain, and infinite dilution.

⁶ E. Lange and J. Monheim, Z. Elektrochem., **35**, 29 (1929), and Z. physik. Chem., [A] **150**, 349 (1930); Wüst and E. Lange, *ibid.*, **116**, 161 (1925); M. Randall and F. D. Rossini, THIS JOURNAL, **51**, 323 (1929). See also F. D. Rossini, Bur. Standards J. Research, **7**, 54 (1931).

(a) L. H. Adams, THIS JOURNAL, 37, 481 (1915); (b) F. Flügel, Z. physik. Chem., 79, 585 (1912); (c) F. Hovorka and W. H. Rodebush, THIS JOURNAL, 47, 1614 (1925); (d) H. Jahn. Z. physik. Chem., 50, 144 (1904); 59, 35 (1907); (e) E. R. Jones and C. R. Bury, Phil. Mag., [7] 3, 1032 (1927), Ref. 1c; (f) G. Karagunis, A. Hawkinson and G. Damköhler, Z. physik. Chem., [A] 150, 433 (1930); (g) H. Menzel, Z. Elektrochem., 33, 68 (1927); (h) A. D. C. Rivett, Z. physik. Chem., 80, 543 (1912); (i) W. H. Rodebush, THIS JOURNAL, 40, 1204 (1918).

⁷ (a) G. N. Lewis and M. Randall, "Thermodynamics," The McGraw-Hill Book Co., New York, 1923, p. 348; (b) M. Randall and A. McL. White, THIS JOURNAL, 48, 2514 (1926).

⁸ W. D. Harkins and R. W. Wampler, THIS JOURNAL, 53, 850 (1931).

 $j = 1 - \vartheta/2 \cdot 1.858m$.

Theoretical values of j, using round values of "a" for various values of $x(=\kappa a)$, and thence $j/zm^{1/2}$ as a function of $zm^{1/2}$ for round values of "a," have been calculated from Equation 113 and Tables II, III and IV of Gronwall, La Mer and Sandved. The coefficients of the terms of the equation are different from those given by Gronwall, La Mer and Sandved, due to the use of Birge's¹⁰ values of $k = 1.3709 \times 10^{-16}$, 0°C. = 273.18°K., $\epsilon = 4.770 \times 10^{-10}$ and the average of Drude's and Wyman's¹¹ determinations of the dielectric constant. The change of three sets of coefficients shown in Table I.

TABLE I								
COEFFICIENTS OF TERMS								
Range of $zm^{1/2}$	1	2	3					
0 to 0.16	3.4480	0.16397	0.07798					
0.16 to 0.41	3.4469	. 16381	. 07785					
0.41 to 1.00	3.4451	. 16355	.07765					

For the most dilute concentrations, then

$$j = 3.4480 \frac{z^2}{10^8 a} \left[\frac{2}{x} - \frac{1}{1+x} + \frac{2}{x^2} \ln(1+x) \right] - 0.16397 \left(\frac{z^2}{10^8 a} \right)^3 10^3 \left[X_3(x) - 6 Y_3(x) \right] - 0.07798 \left(\frac{z^2}{10^8 a} \right)^5 10^5 \left[X_5(x) - 10 Y_5(x) \right]$$
(3)

The first bracket term reduces to $2Y_1(x) - X_1(x)$, where $X_1(x) = 1/(1 + x)$. Values of the bracket terms were calculated, the first, from a reciprocal table and Table II, Gronwall, La Mer and Sandved, the second and third, from Tables III and IV of the same paper, respectively, for round values of "a" at various values of x.

Table II contains the values of j, $j/zm^{1/2}$ and $zm^{1/2}$ resulting from Equation 3. The values of $j/zm^{1/2}$ were plotted as ordinates against $zm^{1/2}$ as abscissas, as in Fig. 1, with the same scales as used in plotting measured $j/m^{1/2} vs. m^{1/2}$ for potassium chloride solutions. By superimposition of the family of the theoretical curves over the measured one, it was estimated that the parameter "a" = 3.6 Å. would coincide with the curve of measured values in the region of lowest concentrations for which the function $j/m^{1/2}$ can be obtained with reasonable accuracy. Values of j, $j/zm^{1/2}$ and $zm^{1/2}$ were calculated for this value of "a" and added to the curve of measured values. These values along with the limiting value of $j/zm^{1/2} = 0.3726$ were used in extrapolating to infinite dilution.

The provisional values of activity coefficients, γ' , were determined by the method of Randall¹² by which

¹⁰ R. T. Birge, Rev. Modern Physics, 1, 1 (1929).

¹¹ (a) P. K. L. Drude, Ann. Physik, **59**, 61 (1896); D = 88.23 - 0.4044t + 0.001035 t^2 ; (b) J. Wyman, Phys. Rev., **35**, 613 (1930); D = 78.54[1 - 0.0046(t - 25) + 0.000008 $(t - 25)^2]$.

¹² M. Randall, This JOURNAL, 48, 2512 (1926).

93		THEORETICAL VALUES OF j , $j/2m^{1/2}$ and $2m^{1/2}$									
44		a in Å.	1	2	3	4	5	6	7	8	3.6
6	x = 0.01	$j \ j/zm^{1/2} \ zm^{1/2}$	$\begin{array}{r} 0.0121932 \\ .3952 \\ .03086 \end{array}$	$\begin{array}{r} 0.0057351\ .3718\ .01543 \end{array}$	0.0037919 .3687 .01029	$\begin{array}{r} 0.0028365\\.3677\\.00771\end{array}$	$\begin{array}{c} 0.0022665\\ .3673\\ .00617\end{array}$	$\begin{array}{c} 0.0018875\\ .3670\\ .00514\end{array}$	$\begin{array}{c} 0.0016172\\ .3669\\ .00441\end{array}$	$\begin{array}{c} 0.0014147\\ 3668\\ .00386\end{array}$	$\begin{array}{c} 0.0031537 \\ .3680 \\ .00857 \end{array}$
CORIDE	x = 0.02	j j/zm ^{1/2} zm ^{1/2}	$.025345 \\ .4107 \\ .06171$.011381 .3689 .03086	.007512 .3652 .02057	.005608 .3635 .01543	. 004477 . 3628 . 01234	. 003727 . 3624 . 01029	003193 . 3622 . 00882	. 002792 . 3619 . 00771	. 006238 . 3639 . 01764
ACTIVITY COBFFICIENTS OF POTASSIUM CHI	x = 0.03	j j/zm ^{1/2} zm ^{1/2}	. 038966 . 4210 . 09256	.017042 .3687 .04628	.011149 .3613 .03086	. 008311 . 3591 . 02314	. 006630 . 3581 . 01851	. 005517 . 3576 . 01543	.004725 .3573 .01322	. 004132 . 3571 . 01157	. 009244 . 3595 . 02571
	x = 0.04	$j \atop zm^{1/2} zm^{1/2}$	$.052622 \\ .4264 \\ .12342$.022512 .3648 .06171	.014672 .3566 .04114	. 010924 . 3541 . 03086	. 008711 3529 . 02468	. 007246 . 3523 . 02057	.006204 .3519 .01763	.005425 .3517 .01543	.012159 .3547 .03428
	x = 0.05	$j \atop zm^{1/2} zm^{1/2}$	$.065298 \\ .4226 \\ .15427$	027847 3610 07714	.018117 .3523 .05142	. 013478 . 3495 . 03857	. 010743 . 3482 . 03086	.008935 .3475 .02571	.007649 .3471 .02204	.006688 .3468 .01928	.015013 .3504 .04285
	x = 0.1	j j/zm ^{1/2} zm ^{1/2}	.127538 .4134 .3085	.052545 .3406 .1543	. 033963 . 3302 . 1029	. 025210 . 3268 . 0771	.020074 .3253 .0617	.016686 .3245 .0514	.014280 .3240 .0441	.012482 .3236 .0386	. 028097 . 3278 . 0857
	x = 0.15	j j/zm ^{1/2} zm ^{1/2}	$.174541 \\ .3771 \\ .4268$.073551 .3178 .2314	. 047609 . 3086 . 1543	.035343 .3055 .1157	$.028141 \\ .3040 \\ .09256$. 023393 . 3033 . 0771	.020018 .3028 .0661	.017498 .3025 .0579	.039392 .3064 .1286
	x = 0.2	$j \atop zm^{1/2} zm^{1/2}$.208936 .3386 .6171	. 091403 . 2962 . 3086	. 059368 . 2886 . 2057	.044132 .2861 .1543	. 035155 . 2848 . 1234	. 029227 . 2842 . 1029	. 025016 . 2838 . 0882	$\begin{array}{c} 021869 \\ .2835 \\ .0771 \end{array}$.049172 .2869 .1714
	x = 0.3	j j/zm ^{1/2} zm ^{1/2}	$.253181 \\ .2735 \\ .9256$. 119348 . 2579 . 4628	. 078275 . 2537 . 3086	. 058332 . 2521 . 2314	. 046524 . 2513 . 1851	.038717 .2510 .1543	. 033151 . 2507 . 1322	. 028988 . 2505 . 1157	.064958 .2526 .2571
, 1932	x = 0.4	j j/zm ^{1/2} zm ^{1/2}	.281187 .2278 1.2342	. 139955 . 2268 . 6171	. 092479 . 2248 . 4117	.069138 .2241 .3086	. 055209 . 2237 . 2468	. 045961 . 2234 . 2057	.039370 .2233 .1763	.034436 .2232 .1543	.076935 .2244 .3428
	x = 0.5	$j \atop zm^{1/2} zm^{1/2}$	$\begin{array}{c} 0.301863 \\ .1958 \\ 1.5427 \end{array}$. 155393 . 2015 . 7714	. 103317 . 2009 . 5142	.077408 .2007 .3857	.061877 .2005 .3086	.051544 .2005 .2571	$044165 \\ 2004 \\ 2204$. 038637 . 2004 . 1928	.086080 .2009 .4285
Dec	x = 0.6	j j/zm ^{1/2} zm ^{1/2}	$\begin{array}{c} 0.318760 \\ .1722 \\ 1.8513 \end{array}$. 167132 . 1805 . 9256	.111627 .1809 .6171	.083732 .1809 .4628	. 067019 . 1810 . 3703	.055848 .1810 .3086	.047868 .1810 .2645	. 041883 . 1810 . 2314	093034 1809 5142

TABLE II

Dec., 1932

$$\log \gamma' = -j/2.3026 - \frac{2}{2.3026} \int_0^m j/m^{1/2} \,\mathrm{d}m^{1/2} + 0.000125 \int_0^m \frac{\vartheta}{m} \,\mathrm{d}\vartheta \qquad (4)$$

Values of ϑ/m were plotted against ϑ and values of the function 0.000125 $\int_0^m \vartheta/m \, d\vartheta$ were determined graphically for round values of concentration. These are given in Table III, along with the values of $-\frac{j}{2.3026}$ $-\frac{2}{2.3026} \int_0^m \frac{j}{m^{1/2}} \, dm^{1/2}$, log γ' and γ' at round values of concentration.



Fig. 1.—Theoretical $j/zm^{1/2}$ vs. $zm^{1/2}$ curves: a, 1 Å.; b, 2; c, 3; d, 4; e, 5; f. 6; g, 7; h, 8 Å.

Table IV contains the values used in the determination of the corrections due to the heat of dilution.

$$\log \gamma = \log \gamma' - 27.754 \int_0^m \frac{1}{m} \,\mathrm{d}x$$

where $x = -\overline{L}_{1(298)}y + (\overline{C}_{p_1} - \overline{C}_{p_1^0})z$, y and z being temperature functions defined by Lewis and Randall.^{7a} (Note other use of x and z.) The values of \overline{L}_1 , the partial molal heat content of water in potassium chloride solutions at 25°, are by Lange and Monheim, those of the partial molal heat capacity for the solvent less that of pure water are by Randall and Rossini.⁵ The values of the function $-27.754 \int_0^m (1/m) dx$ were determined graphically and are included in Table IV along with the corrected values of $-\log \gamma$ and γ .

Vol. 54

Dec., 1932

TABLE III

PROVISIONAL VALUES OF ACTIVITY COEFFICIENT OF POTASSIUM CHLORIDE IN AQUEOUS Solutions

			- 2	4.0		
			2.3026	1.25×10^{-4}		
m	m ^{1/2}	$-\frac{j}{2.3026}$	$\int_0^m j/m^{1/2} \mathrm{d}m^{1/2}$	$\int_0^{m} \vartheta/m \mathrm{d}\vartheta$	$\log \gamma'$	γ'
0	0	0	0		0	1
0.001	0.0316	-0.00489	-0.01000		-0.01489	0.9663
.002	.0447	00678	01402		02080	.9532
.005	.0707	01031	02174	0.00001	– .03204	. 9289
.01	. 1000	01392	03020	.00002	04410	.9034
.02	. 1414	01848	04129	. 00003	05974	.8715
.05	. 2236	02586	06153	.00008	08731	.8179
. 1	.3162	03203	08096	.00015	11284	.7712
.2	. 4472	03836	10517	.00029	14324	. 7358
.35	.5916	04376	12815	.00057	17134	.6740
. 5	.7071	04720	14436	.00071	19085	.6444
.75	. 8660	05089	16424	.00103	21410	.6108
1.0	1.0000	05346	17917 ·	.00137	23126	. 5871
1.25	1.1180	05496	19127	.00169	24454	.5695
1.5	1.2247	05569	20138	.00201	25506	. 5 559

Discussion of Results

Gronwall, La Mer and Sandved probably had a network plot of j as a function of x for the two sets of parameters "a" and $z\kappa$ for the solution of their Equation 113. Their treatment would undoubtedly lead to satisfactory values of $\vartheta_{(obs.)} - \vartheta_{(calcd.)}$, and thus of "a," if applied in the range for which the experimental data and the assumptions underlying the theory are sufficiently accurate, and one could calculate log γ in the very dilute range by Equation 2. However, if one were using the freezing point data for the determination of activity coefficients, the experimental data would be cast into the $j/zm^{1/2}$ vs. $m^{1/2}$ form for the range above which the assumptions of the theory are not expected to be valid, so this should prove the simpler method. The theoretical $j/zm^{1/2}$ vs. $zm^{1/2}$ curve can of course be used for any symmetrical valence type salt.

A plot of the function $-27.754 \int_{0}^{m} (1/m) dx$ against molality shows that the correction for the heat of dilution lowers the activity coefficient at concentrations below about 0.12 M for potassium chloride.

The data of Harned for the cell

Ag, AgCl(s), KCl(m), K_xHg, KCl(0.1 M), AgCl(s)Ag

on which his values of the activity coefficient were based have been put into the form $^{13}\,$

$$\log \gamma + \text{const.} = -\frac{E}{0.11831} - \log m \pm$$

¹⁸ M. Randall, Trans. Faraday Soc., 23, 502 (1927).

Corrections Due to Heats of Dilution and Final Values of the Activity Coefficients of Potassium Chloride in Aqueous Solutions at 25°											
m	ð	\overline{L}_1	$\overline{C}_{p_1} - \overline{C}_{p_1}^{0}$	$-y\overline{I}_{A}$	$+z(\overline{C}_{p_1}-\overline{C}_{p_1}^0)$	x	$\frac{1}{m}$	$-27.754\int_0^m \frac{1}{m}\mathrm{d}x$	-log γ'	$-\log \gamma$	r
0.001									0.01489		0.9663
.002 .005									.02080		.9552
.01	0.037	-0.0027	-0.0002	+0.0000018	-0.0000017	+0.0000001	100	-0.00006	.04410	0.04416	. 9033
. 02	.071	0061	0004	+ .00000041	00000034	+ .00000007	50	00017	.05974	.05991	.8711
.05	.171	0161	0009	+ .00000109	00000077	+ .00000032	20	00044	. 08731	.08775	8171
.1	.342	0223	0019	+ .00000152	00000169	00000017	10	00026	. 11284	.11310	7707
.2	.677	+ .0110	009	00000076	00000819	00000895	5	+ .00148	.14324	.14176	. 7383
.35	1.330	+ .27	020	0000192	0000194	0000386	2.86	+ .00454	.17134	.16680	.6811
. 5	1.655	+ .76	034	0000547	0000336	0000883	2	+ .00780	19085	.18305	.6561
. 75	2.464	+2.23	063	0001658	0000668	0002326	1.33	+ .01262	.21410	.20148	.6288
1.	3.265	+3.95	099	0003031	0001109	0004140	1	+ .01845	. 23126	21281	.6126
$1 \ 25$	4.066	+5.95	141	000471	0001677	000639	0.8	+ .02407	.24454	.22047	. 6019
1.5	4.866	+8.22	187	000671	0002340	000905	. 667	+ .02948	.25506	.22558	. 5949

TABLE IV

Vol. 54

and plotted as abscissas against $m^{1/\epsilon}$ as ordinate, and compared with a plot of log γ vs. $m^{1/\epsilon}$ from freezing point measurements. It is scarcely possible to superimpose as much of the first curve as that between two concentrations on the second curve. In view of the forthright thermodynamic character of the freezing point method involving, besides inaccuracy of data, only the assumption¹⁴ of constancy of \overline{C}_{p_1} - $\overline{C}_{p_1^0}$ it appears that the cell reaction is not purely the one postulated. If temperature coefficients of the cell reaction were available, a test of the purity of reaction could be made by comparison of the ΔH 's with thermochemical values.

Summary

1. The activity coefficients of potassium chloride in aqueous solutions at 25° have been determined from freezing point measurements including the recent ones by Jones and Bury and by Karagunis, Hawkinson and Dam-köhler. Corrections for the heats of dilution have been made.

2. Theoretical values of j, $j/zm^{1/2}$ and $zm^{1/2}$ for symmetrical valence type salts have been calculated from the extension of the Debye-Hückel theory by Gronwall, La Mer and Sandved and offer a satisfactory guide for extrapolating from the region where the function $j/zm^{1/2}$ becomes difficult or impossible to measure accurately, to infinite dilution.

UNIVERSITY, VIRGINIA

Since this paper was written Saxton and Smith [THIS JOURNAL, 54, 2626 (1932)] have reported the results of their accurate measurements of the boiling points of potassium chloride solutions. They use the same values of $\overline{C}_{p_1} - \overline{C}_{p_1^0}$ at 25° and slightly different values of L_1 in correcting their activity coefficients to 100°. It is interesting to note that by assuming that $\overline{C}_{p_2} - \overline{C}_{p_2^0}$ is also independent of temperature, they find the activity coefficients at 25° by Harned^{1d} lead to values at 100° in good agreement with their values from boiling point measurements.

¹⁴ Undoubtedly this assumption is not strictly valid. From the data assembled by Awbery ["International Critical Tables," Vol. V, p. 115] and Rossini's approximate equations for $\overline{C}_{p_1} - \overline{C}_{p_1^0}$ (Ref. 5), one finds for 0.5551 M KCl $-(\overline{C}_{p_1} - C_{p_1^0})$ at 18, 21.5 and 25° equal to 0.076, 0.065 and 0.065, for 1.110 M KCl, 0.155, 0.155 and 0.167, and for 1.665 M KCl, 0.203, 0.203 and 215 cal. mole⁻¹ deg.⁻¹, respectively. Thus over this smaller range of temperature $\overline{C}_{p_1} - \overline{C}_{p_1^0}$ is practically independent of the temperature, though the values at 25° do not agree with those derived without using equations by Randall and Rossini. The value of the correction is believed not to be seriously affected by this uncertainty.